

## **APPENDIX 2: CALCULATION OF SPOT REACH USING A PROBABILITY MODEL**

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### **1. INTRODUCTION**

Different forms of analyses carried out on the same data should yield the same values for a specific result type. Moreover, related result types should yield values that stand in a logical relationship to one another. When these conditions are not met, it can create confusion for the user. For this reason a continual focus on definitions and methods of calculation is warranted.

There does not seem to be much of a problem in arriving at a uniform reporting with respect to the determination of an item such as audience rating. However, guaranteeing a similar logical coherence for the result "net reach" is more problematic. In fact, these two result types are related; the viewing frequency distribution and the related reach for a series of spots yield a number of GRPs, which is equal to the sum of audience ratings. It does not seem that determining spot reach should be anymore difficult that determining audience rating, all the more since the spot reach after one broadcast should be equivalent to the audience rating for that broadcast.

The problems that arise are a consequence of choices made in arriving at a definition. There is a single, generally accepted definition of audience rating, although it is perhaps less transparent than it may appear. As will become apparent, however, there are several definitions of net reach in use, leading to differences in methods of calculation and, consequently, to differing results.

In the Dutch television audience research, a definition of spot reach has been chosen that is logically consistent with the definition of audience rating.

### **2. SPOT REACH AND TIME SHIFTED VIEWING**

In the audience research, spot reach is calculated via the probability model. This is done to maintain the relationship between audience ratings, the number of GRPs and contact frequency, even when audience rating is measured on a continuous basis (viewing duration method). In the sections that follow, we examine how this relationship comes into being and how calculating spot reach using the probability model can guarantee this relationship.

From January 1, 2008, time shifted viewing behaviour is included in the total viewing. This means that audience ratings of over 100% could be achieved, at which point the definition of viewing possibilities and the spot reach formula will not be applicable in analyses of data including time shifted viewing. Consequently, the spot reach formula can only be calculated based on the viewing moment of the time shifted viewing.

In contrast to the treatment of time shifted viewing in other result types, in the calculation of spot reach, time shifted viewing is not assigned to the original broadcast moment. When the delayed viewing of a spot takes place at a specific time, it is assumed that this viewing moment creates a new contact moment.

Thus, this result type must always be calculated based on the viewing moment, using the WVG file in combination with the UGK file.

Figure 6.1: Example of the treatment of time shifted viewing by Programs and Time Slots.

Programs/Time slots Absolute number of viewers (gross)	Day 0 Saturday	Day 3 Tuesday	Day 5 Thursday
Time slot 18:30-19:30 Program a, Block, Program b	100	5	5
	↑ Viewers Reported on		
<b>Totaalscore Currency</b>			
Program a	110		
Block	110		
Program b	110		

Figure 6.2: Example of the treatment of time shifted viewing by Spots and TV Advertising campaigns.

**Campaign X with 2 spots:**

- A. 1-7, NL1, 20.00.00-20.00.59
- B. 1-7, RTL4, 21.00.00-21.00.59

**Viewers:**

Resp 1: seen A 1-7, NL1, 20.00.00-20.00.23 = 24"  
B 1-7, RTL4, 21.00.00-21.00.11 = 12"

Resp 2: seen B 1-7, RTL4, 21.00.00-21.00.59 = 60"

And spot B also time shifted viewed two days after broadcast:

-> This becomes a new contact for calculation purposes, we call it

seen B time shifted viewed 3-7, RTL4, 16.00.00-16.00.29 = 30"

Resp 3: has not seen the campaign

**Method:**

In order to bring calculation and reporting together we need the following information:

Spot	A	B	B-TSV
<b>Resp1</b>	<b>0.4</b>	<b>0.2</b>	<b>0.0</b>
<b>Resp2</b>	<b>0.0</b>	<b>1.0</b>	<b>0.5</b>
<b>Resp3</b>	<b>0.0</b>	<b>0.0</b>	<b>0.0</b>

### Calculation according to calculation rules:

(Weighting is omitted in this example)

	A (1-7, NI1)			B (1-7, RTL4)			B-TSV (3-7, RTL4)		
	viewing moment	p1	q1	viewing moment	p2	q2	viewing moment	p3	q3
Resp 1	24	0.4	0.6	12	0.2	0.8	0	0	1
Resp 2	0	0	1	60	1	0	30	0.5	0.5
Resp 3	0	0	1	0	0	1	0	0	1
	24			72			30		
100(%)*sum/3	<b>Sec A</b>	13.3%		<b>Sec B</b>	40.0%		<b>Time shifted</b>	16.7%	
Rating	0.4*0.33 = 13.0%			0.2*0.33+1*0.33 = 40.0%			<b>0.5*0.33 = 17.0%</b>		

Sum GRPs = 13%+40%+17% = 70%

### Number of spots viewed A, B en B TSV (Total viewing, 1+):

	f(0,1)	f(1,1)	f(0,2)	f(1,2)	f(2,2)	f(0,3)	f(1,3)	f(2,3)	f(3,3)
	q1	p1							
Resp 1	0.6	0.4	0.48	0.44	0.08	0.48	0.44	0.08	0
Resp 2	1	0	0	1	0	0	0.5	0.5	0
Resp 3	1	0	1	0	0	1	0	0	0
	2.6	0.4	1.48	1.44	0.08	1.48	0.94	0.58	0
100(%)*sum/3	86.7%	13.3%	49.3%	48.0%	2.7%	<b>49.3%</b>	<b>31.3%</b>	<b>19.3%</b>	<b>0.0%</b>

Net reach =  $f(1,3)+f(2,3)+f(3,3) = 31.3\%+19.3\%+0.0\% = 50.7\% = 1-49.\% = 1- f(0,3)$

From the frequency distribution we have =  $1 \times f(1,3) + 2 \times f(2,3) + 3 \times f(3,3) = 1 \times 31.3\% + 2 \times 19.3\% + 3 \times 0.0\% = 70\% = \text{sum GRP's}$

### Reach building

Total respondents

Frequency class	0	1	2	3	Sum (horizontaal)
<b>Spot in the schedule</b>					
A	0.867	0.133			1
B	0.493	0.48	0.0267		1
B TSV	0.493	0.313	0.1933	0	0.973
<b>Grps</b>	0	0.3	0.4	0	<b>0.7</b>

### Rapporting (total, by each spot)

	SPOT A		SPOT B		TSV SPOT B- TSV		CURRENCY SPOT A, B and B-TSV
	p1	q1	p2	q2	p3	q3	q1 * q2 * q3
Rp1	0.4	0.6	0.2	0.8	0	1	0.48
Rp2	0	1	1	0	0.5	0.5	0
Rp3	0	1	0	1	0	1	1
Sum	0.4	2.6	1.2	1.8	0.5	2.5	1.48
Sum/3 (=rating %)	0.133	0.867	0.400	0.600	0.167	0.833	0.493
Net Reach		13.3%		40.0%		16.7%	50.7%
Not reached		86.7%		60.0%		83.3%	49.3%

	SPOT A		SPOT B		SPOT B- TSV		OVERNIGHT SPOT A and B
	p1	q1	p2	q2	p3	q3	q1 * q2
Rp1	0.4	0.6	0.2	0.8	0	1	0.48
Rp2	0	1	1	0	0.5	0.5	0
Rp3	0	1	0	1	0	1	1
Sum	0.4	2.6	1.2	1.8	0.5	2.5	1.48
Sum/3 (= rating kdh%)	0.133	0.867	0.400	0.600	0.167	0.833	0.493
Net Reach		13.3%		40.0%		16.7%	50.7%
Not reached		86.7%		60.0%		83.3%	49.3%

- Total campaign can be reported as follows.

TSV	TOTAL VIEWING	OVERNIGHT
1+Reach	50.7%	50.7%
16.7%		

- Reach after Spot A

	Spot A	
	p1	q1
Rp1	0.4	0.6
Rp2	0	1
Rp3	0	1
Som	0.4	2.6
Sum/3 (= rating%)	0.133	0.867
Net Reach		13.3%
Not reached		86.7%

After Spot A =  $Rp1*0.4 + Rp2*0.0 + Rp3 * 0.0 = 0.4$   
 GRPs = Sum/3 (= rating%) =  $0.4/3 = 0.133$

	TOTAL VIEWING	OVERNIGHT	TSV
Reach after spot A. (1-7, NL1, 20.00.00-20.00.59)	13.3%	13.3%	
0.0%			

- Reach after Spot B

	SPOT A		SPOT B		SPOT B- TSV		SPOT B and SPOT B-TSV
	p1	q1	p2	q2	p3	q3	q2*q3
<b>Rp1</b>	0.4	0.6	0.2	0.8	0	1	0.8
<b>Rp2</b>	0	1	1	0	0.5	0.5	0
<b>Rp3</b>	0	1	0	1	0	1	1
<b>Som</b>	0.4	2.6	1.2	1.8	0.5	2.5	1.48
<b>Sum/3 (= rating%)</b>	0.133	0.867	0.400	0.600	0.167	0.833	0.493
<b>Net Reach</b>		13.3%		40.0%		16.7%	50.7%
<b>Not reached</b>		86.7%		60.0%		83.3%	49.3%

By the calculation of the reach of spot B and B Time Shifted Viewed we treat B and B-TSV as separate events:

$$Rp1*0.4 + Rp2*0.0 + Rp3 * 0.0 \text{ (reach SPOT A)} + Rp1 0.2 + Rp2 1 + Rp3 0.0 \text{ (reach SPOT B)} + Rp1 0.0 + Rp2 0.5 + Rp3 0.0 \text{ (reach SPOT B-TSV)} = 0.4 + 1.2 + 0.5 = 2.1$$

$$GRPs = \text{Sum}/3(=kdh\%) = 2.1/3 = 0.70$$

**Net Reach = 50.7%**

	TOTAL VIEWING	OVERNIGHT	TSV
Reach after spot B. (1-7, RTL4,21.00.00-21.00.59)	50.7%	50.7%	
40.0%			

#### TOTAL VIEWING

	GRPs	Total	1+	2+
<b>After A</b>	13.3	13,3	13.3	
<b>After B</b>	70.0	50.7	31,3	19.3

### 3. AUDIENCE RATING AND VIEWING PROBABILITIES

Measuring audience levels is done by observing at specific moments - measurement moments - whether a person is watching a broadcast - viewing moments. It is commonly accepted that saying a program has an x% audience rating means that x% of people have watched it. However, there is no basis for such a formulation in the definition. What does "watched it" actually mean. In theory, this can vary from a single viewing moment to watching the entire program.

The method of calculating audience rating takes into account the length of time a respondent has watched a broadcast: two respondents each see half of a program or one watches the entire program and the other does not see any of it; both situations yield a 50% audience rating. This is because calculation is carried out - for good reasons - on viewing and measurement moments.

The ratio of viewing moments and measurement moments for one person is a relative viewing volume that we will hereafter refer to as viewing probability. The term probability is appropriate because we are dealing with a statistical probability that a measurement moment will be a viewing moment. If the measurement moments form a representative sample of all possible measurement moments in the universe, formulas for calculating probability, and in particular, estimation formulas can be applied.

In the following formula, the viewing probability for a single respondent is represented by the letter p. The letter K represents all viewing moments k for that person in a broadcast, and the letter M for all of the measurement moments m. Thus, for a person i:

$$p_i = \frac{\sum k_i}{\sum m_i} = \frac{K_i}{M_i}$$

Furthermore, we can introduce the letter q<sub>i</sub> to represent the compliment of p<sub>i</sub>, the probability of non-viewing:

$$q_i = 1 - p_i = \frac{M_i - K_i}{M_i}$$

The precision of the measurement of viewing probability is dependent on M, the frequency of measuring moments, which can be compared to the size of the sample. There are several alternatives available; but in this discussion we will distinguish between the crude "snap shot" method and the more precise "total viewing time" method.

In practice, the lowest frequency used in television research is one measurement moment per minute, the snap shot method. Using this method, one station can be assigned per minute for each viewer. The possible number of values for p for a time interval, program or spot is directly proportional to the number of measurement moments. By increasing the number of measurement moments, a more precise value can be calculated for p. This is comparable to extending the decimal places for a number. If we increase the number of measurement moments to the technical maximum (one per second) then we arrive at the viewing time method.

In both cases, the definition of audience rating is:

$$kdh = \frac{\sum_{i=1}^n p_i}{n} = \frac{1}{n} \sum_{i=1}^n \left( \frac{K_i}{M_i} \right)$$

Where n = the number of respondents in the sample.

As a result of this definition, a respondent is included in the audience rating on the basis of viewing probability rather than as 0 or 1.

The viewing time method will produce a more precise figure than the snap shot method because it is based, as it were, on a larger sample and will produce values that are more precise.

#### 4. CONSEQUENCES FOR THE DEFINITION OF SPOT REACH

One important conclusion that can be drawn from the previous discussion is that it is appropriate, when considering audience ratings, to use viewing probability to take account of "partial viewing". Instead of including in the calculation only those respondents who watched an entire event (program, quarter or spot), each observed viewing moment is counted. This is a shift from a dichotomous approach characterised by two possibilities: yes or no, to one offering a (virtually) continuous scale of possibilities.

It would seem appropriate to apply a similar approach to spot reach, as it is so closely related to audience rating. However, this is made difficult by the dichotomous character one intuitively assigns to reach. While the term 'audience rating' suggests continuity (rating on a scale of values), the same cannot be said for reach: one is either reached or not reached, not reached a little bit or reached a lot. The association of a dichotomous principle with reach is carried forward in the definitions of program and station reach in use.

The fact that a similar dichotomous principle is not applied to spot reach is due to the previously mentioned close relationship between this result type and a number of other important result types, namely contact frequency, audience rating and the number of GRPs.

Intuitively,

- a. multiplying spot reach by the average contact frequency should produce the number of GRPs, which, in turn, is the sum of the spot audience ratings (i.e., the audience rating for the commercial minute).
- b. because of the dichotomous nature of the concept contact - there is contact or there is no contact - a contact frequency will be expressed as a whole number.

The definition of spot reach should satisfy both of these intuitive assumptions. While the resulting problems hold for series of spots of whatever size, they are most clearly seen in the case of a single spot. The average contact frequency for viewers is equal to 1, according to point b, and the number of GRPs equal to the audience rating (of the commercial minute). However, according to point a, spot reach should be the same as the audience rating, which is measured on a continuous scale.

The following hypothetical example illustrates how a definition of reach, in this case program reach, fails to satisfy the above relationships.

The number of viewers during an advertising minute (duration of measure = 60 seconds) can be distributed according to the duration of viewing as follows:

(A) viewing time K	(B) %	(C = A/60) p = K/M	(B x C) contribution to audience rating
20 sec.	3%	$\frac{1}{3}$	1%
30 sec.	4%	$\frac{1}{2}$	2%
60 sec.	6%	1	6%
			<u>9%</u>

This results in an audience rating of 9%. A program reach with a 50 % minimum viewing percentage includes every respondent with a viewing time of 30 seconds or longer. In this example, this is:  $4 + 6 = 10\%$ .

Preceding from point a above produces the following inconsistency:

$$\begin{array}{rclcl} \% \text{ reach} \times & \text{average contact frequency} & = & & \text{number of GRPs} \\ = & \text{audience rating} & & & \\ 10 & \times & 1 & = & 10\% \neq 9\% \end{array}$$

The inequality that arises in this example is a consequence of the friction between the two basic assumptions: the continuous character of the audience rating as against the dichotomous character of the reach. Everyone for whom as much as a single viewing moment has been observed is included in audience rating, but not in program reach.

A reach value cannot be considered independently of the reach criteria used to generate it. As we have seen, there are a number of reach criteria in use in audience research, and many more are possible. In order to be able to calculate spot reach in a consistent manner, it is necessary to turn to one of the various alternatives that underlie the models used in media planning in the theory of media reach. One such model is the individual probability model, which underlies the standard method of calculating spot reach.

## 5. SPOT REACH ACCORDING TO THE PROBABILITY MODEL

As soon as at least one measurement moment produces a viewing moment for a respondent, there are consequences for the determination of audience rating. There is a reach principle readily available for the desired relationship between spot reach/contact frequency and audience rating; it is based on a minimum criterion: "some" of a broadcast seen. For every definition of reach, a respondent contributes to reach as soon as the criteria in the definition are satisfied. In this case, it is when at least one viewing moment is found.

The next question, then, is, "What is the contribution involved?" In the case of the snap shot method, the answer is simple. The single moment of measurement in the advertising minute produces a viewing moment or it doesn't. If it does, then this counted as a contact and, if the respondent has not yet had any contacts, this included in the net reach. This approach has the advantage that the number of contacts will always be a whole number. In the Dutch television audience research, however, we have opted for viewing time method (see § 2). This method will produce more precise information than the single snap shot, but will not necessarily result in a whole number of contacts. This, in turn, can be a problem for, for example, the calculation of contact frequency.

To deal with this difficulty, an estimate can be made of the probability that a random measurement in an advertising minute will produce a contact (as a whole number). In order to maintain the minimum criterion, this estimate is made based on the viewing time method, thus on all of the measurement seconds in the minute. The following quantities are calculated per respondent I for the minute k:

$$p_i(k) = \frac{\text{number of viewing moments}}{\text{number of measurement moments}} = \frac{\text{viewing}}{60}$$



$$q_i(k) = 1 - p_i(k) = \frac{\text{number of not-viewing moments}}{\text{number of measurement moments}} = \frac{\text{not-viewing}}{60}$$

One of the basic tenets of probability theory is the multiplication theorem: If A and B are two independent events, then the probability of both A and B occurring is equal to the product of the probabilities of each event occurring separately:

$$p(A \& B) = p(A) \times p(B)$$

If we introduce the symbol  $c$  for the number of contacts in a campaign with  $n$  broadcasts, then  $f_i(c, k)$  represents the contact frequency for a respondent after  $k = 1, 2, \dots, n$  broadcasts. On the basis of the multiplication theorem:

$$f_i(c, k) = f_i(c-1, k-1) \times p_i(k) + f_i(c, k-1) \times q_i(k)$$

Applying this formula produces an iterative calculation, with the initial values:

$$f_i(1, 1) = p_i(1)$$

$$f_i(0, 1) = q_i(1)$$

This procedure is illustrated in the following example of the calculation of the probability distribution of contact frequency with  $k = 3$ :

$$f_i(0, 3): \text{probability of 0 after 3: (probability of 0 after 2) * } q$$

$$f_i(1, 3): \text{probability of 1 after 3: (probability of 0 after 2) * } p + \text{(probability of 1 after 2) * } q$$

$$f_i(2, 3): \text{probability of 2 after 3 is equal to: (probability of 1 after 2) * } p + \text{(probability of 2 after 2) * } q$$

$$f_i(3, 3): \text{probability of 3 after 3 is equal to: (probability of 2 after 2) * } p$$

When the calculation has been completed through  $k = n$  for a respondent with the weight factor  $g_i$ , the respondent's contribution to the total of a frequency group  $c$  is counted as:  $g_i f_i(c, n)$ .

After division by the sum of the respondent weightings, the frequency distribution obtained in this manner produces a proportional representation that can be converted to percentages. The net reach can then be determined by summing all the percentages belonging to  $c \geq 1$ .

## 6. AN EXAMPLE OF THE CALCULATION FORMULA

Figure 6.2. presents a simple example of the calculation for 5 respondents and 2 broadcasts. The weighting of respondents is not included in the example, but this is simple to implement.

	broadcast 1			broadcast 2			
	number of measure moments						
	60			60			
		viewing moment	$p(1)$	$q(1)$	viewing moment	$p(2)$	$q(2)$
respondent 1	24	0.40	0.60	3	0.05	0.95	
respondent 2	0	0.00	1.00	15	0.25	0.75	
respondent 3	12	0.20	0.80	6	0.10	0.90	
respondent 4	0	0.00	1.00	0	0.00	1.00	
respondent 5	6	0.10	0.90	60	1.00	0.00	

total	42	84
audience rating	<b>14.0%</b>	<b>28.0%</b>

number of broadcasts seen:	$f(0,2)$					$f(1,2)$	$f(2,2)$
	$f(0,1)$	$f(1,1)$	$f(0,1)q(2)$	$f(0,1)p(2)$	$f(1,1)q(2)$	$a+b$	$f(1,1)p(2)$
respondent 1	0.60	0.40	0.57	0.03	0.38	0.41	0.02
respondent 2	1.00	0.00	0.75	0.25	0.00	0.25	0.00
respondent 3	0.80	0.20	0.72	0.08	0.18	0.26	0.02
respondent 4	1.00	0.00	1.00	0.00	0.00	0.00	0.00
respondent 5	0.90	0.10	0.00	0.90	0.00	0.90	0.10
sum			3.04			1.82	0.14
100 (%) x sum / 5			<b>60.8%</b>			<b>36.4%</b>	<b>2.8%</b>

Figure 6.2: Example of the calculation of spot reach and contact frequency.

In the example, 36.4% saw one broadcast and 2.8% both broadcasts, so that the net reach comes to  $36.4 + 2.8 = 39.2\%$  or, using the null reach,  $100 - 60.8 = 39.2\%$ .

The sum of the GRPs is  $14.0 + 28.0 = 42.0\%$ .

From the frequency distribution, it follows that:  $1 \times 36.4 + 2 \times 2.8 = 42.0$ , thus satisfying the requirement of agreement between audience ratings and frequency distribution.

The calculation produces a mathematically balanced result. However, the significance of the calculation is more than mathematical in nature. It is a calculation of probability, a statistical calculation, to estimate the projection of *all* respondents based on *all* theoretical moments. It is therefore sensible to count fractional contacts for a separate respondent (something that in practice is not possible) as a contribution to as accurate an estimated average value as possible.